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Nonlinear effects in microwave photoconductivity of two-dimensional electron systems

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Abstract
We present a model for microwave photoconductivity of two-dimensional electron systems in a magnetic field which describes the effects of strong microwave and steady-state electric fields. Using this model, we derive an analytical formula for the photoconductivity associated with photon- and multi-photon-assisted impurity scattering as a function of the frequency and power of microwave radiation. According to the developed model, the microwave conductivity is an oscillatory function of the frequency of microwave radiation and the cyclotron frequency which becomes zero at the cyclotron resonance and its harmonics. It exhibits maxima and minima (with absolute negative conductivity) at microwave frequencies somewhat different from the resonant frequencies. The calculated power dependence of the amplitude of the microwave photoconductivity oscillations exhibits pronounced sublinear behaviour similar to a logarithmic function. The height of the microwave photoconductivity maxima and the depth of its minima are nonmonotonic functions of the electric field. The possibility of a strong widening of the maxima and minima due to a strong sensitivity of their parameters on the electric field and the presence of strong long-range electric-field fluctuations is pointed to. The obtained dependences are consistent with the results of the experimental observations.

1. Introduction

Transport properties of two-dimensional electron systems (2DESs) subjected to a transverse magnetic field were extensively studied in the late 1960s and early 1970s. To the best of our knowledge, the first paper on this topic was published by Tavger and Erukhimov [1]. In this paper, the dissipative conductivity (diagonal component of the conductivity tensor) associated with the tunnelling between the Landau levels (LLs) accompanied by the impurity scattering of electrons was calculated as a function of the electric and magnetic field. The dissipative current–voltage characteristic obtained considering the impurity scattering in the Born approximation [1] is given by an non-analytical dependence $j \propto E^{-2}\exp(-E_c^2/2E^2)$, where E is the electric field, $E_c = \hbar\Omega_c/eL$, $\hbar$ is the Planck constant, Ω_c and L are the cyclotron frequency and quantum Larmor radius, respectively, and e is the electron charge. Later, it was shown [2] that more strict consideration of the impurity scattering leads to the 'restoration' of the Ohm law, so that $j \propto E$ at $E < E_b = \hbar\Gamma/eL$, where $\hbar\Gamma$ is the LL broadening, and $j \propto E^{-1}$ when $E_b < E \ll E_c$. Pokrovsky *et al* [3] demonstrated that the inclusion of the processes of the electron inter-LL tunnelling via bound impurity states can give rise to an exponential dependence similar to that obtained in [1] but with a smaller characteristic field E_c. Theoretical studies of the transport in a 2DES irradiated with microwaves or under non-equilibrium conditions associated with intraband or intersubband optical excitation were carried out in [4–6]. In particular, it was predicted by Ryzhii [4] (see also [7]) that microwave radiation with the frequency Ω somewhat exceeding the cyclotron frequency or its harmonics $\Lambda\Omega_c$, where Λ is an integer, can result in absolute negative conductivity (ANC) in a 2DES. Nonlinear microwave photoconductivity associated with the effect of photon-assisted impurity scattering of electrons was studied theoretically in [8–10]. In particular, it was demonstrated that the multi-photon processes (both virtual and real) can significantly influence the dissipative and Hall components of the conductivity tensor. Since current–voltage characteristics with ANC inevitably exhibit ranges with negative differential conductivity (NDC), it was clear that time that uniform states of a 2DES with ANC can be unstable [11].

An interest in theoretical studies of non-linear transport in 2DESs has revived after the observation of the breakdown of the quantum Hall regime [12] (see, for example, the articles by Heinonen *et al* [13], Balev [14], Chaubet *et al* [15, 16], and Komiyama *et al* [17, 18], and the review by Nachtwei [19]).

Recent observations of vanishing electrical resistance/conductance in 2DESs caused by microwave radiation [20–23], have stimulated extensive efforts to clarify the nature of the uncovered effects and triggered a new surge of theoretical papers (see, for example, [24–33]). The occurrence of the so-called zero-resistance/conductance states is primarily considered [24–27, 32] as a manifestation of the effect of ANC associated with the photon-assisted impurity scattering [4, 7, 26, 32], possibly, complicated by the scattering processes involving acoustic phonons [30–33]. The role of multi-photon-induced scattering processes was evaluated in [28, 29] using models similar to that considered earlier [8, 9]. Vavilov and Aleiner [32] have developed a general approach based on the quantum Boltzmann equation in the quasi-classic approximation which provides a description of non-linear effects in a 2DES. In this paper, we use a more simple and transparent model of the microwave conductivity bearing in mind the goal to obtain explicit analytical formulae describing non-linear effects, namely, formulae for the dependences of the photoconductivity maxima and minima on the microwave power and the electric field (i.e., on the ac and dc fields). In particular, we demonstrate that the magnitude of the microwave photoconductivity maxima and minima is a non-monotonic function of the microwave power and the electric field. It is also shown that an increase in the microwave power and/or the electric field leads to a marked shift of the maxima and minima and an increase in their width. Due to a high sensitivity of the microwave photoconductivity to the local electric field, the observable characteristics of the 2DES can be essentially affected by long-range electric-field fluctuations.

The obtained results shed light on some features of the effects observed experimentally.

After this introduction, in section 2, we write down a general formula for the dissipative current based on the notion that this current is associated with the spatial displacement of the electron Larmor orbit centres caused by the photon-induced impurity scattering processes. The probability of these processes is a function of the net dc electric field (including both the applied and Hall components) and the ac microwave electric field. This probability is presented as a sum of the terms corresponding to the participation of different numbers of real photons [8, 9]. Such a technique allows us to bypass the diagram (perturbation) summation. In section 3, we calculate the microwave photoconductivity in a relatively low dc electric field (Ohmic regime) as a function of the microwave frequency and power. Section 4 deals with the calculation of the microwave photoconductivity in a strong dc electric field when the latter substantially affects the scattering processes. In section 5, we consider the effect of microwave radiation on intra-LL impurity scattering. Section 6 deals with the discussion of the obtained results and its relevance to the pattern of the experimental observations.

2. General equations

At low temperatures $T \ll \hbar\Omega_c$ and low electric fields $E \ll E_c$, the dissipative dark current (without irradiation) is associated mainly with the electron transitions within the same LL. Under the microwave radiation, the inter-LL electron photon-assisted transition can markedly contribute to the dissipative current. The microwave radiation can also affect the intra-LL scattering processes. Therefore, the microwave photocurrent, i.e., the variation of the dissipative current caused by irradiation, can be presented as

$$j_{\rm ph} = j_{\rm ph}^{\rm (inter)} + j_{\rm ph}^{\rm (intra)}, \tag{1}$$

where $j_{\rm ph}^{\rm (inter)}$ is the contribution associated with the electron transitions between the LLs with different indices stimulated by microwave radiation and the second term on the right-hand side of equation (1) is the variation of the intra-LL component of the dissipative current.

For the probability of the transition between the (N, k_x, k_y) and $(N', k_x + q_x, k_y + q_y)$ electron states in the presence of the net dc electric field $\mathbf{E} = (E, 0, 0)$ perpendicular to the magnetic field $\mathbf{H} = (0, 0, H)$ and the ac microwave field $\mathbf{E}_\Omega = (\mathcal{E}e_x, \mathcal{E}e_y, 0)$ polarized in the 2DES plane (e_x and e_y are the components of the microwave field complex polarization vector), we will use the following formula obtained on the basis of the interaction representation of the operator of current via solutions of the classical equations of electron motion [8, 9]:

$$\begin{aligned} W_{N,k_x,k_y;N',k_x+q_x,k_y+q_y} &= \frac{2\pi}{\hbar}\sum_M \mathcal{N}_{\rm i}|V_q|^2|Q_{N,N'}(L^2q^2/2)|^2 \\ &\quad\times J_M^2(\xi_\Omega(q_x,q_y))\delta[M\hbar\Omega + (N-N')\hbar\Omega_c + eEL^2q_y]. \end{aligned} \tag{2}$$

Here N is the LL index, k_x and k_y are the electron quantum numbers, (q_x and q_y) are their variations due to photon-assisted impurity scattering, $q = \sqrt{q_x^2 + q_y^2}$, $e = |e|$ is the electron charge, $\mathcal{N}_{\rm i}$ is the impurity concentration, and $V_q \propto q^s \exp(-d_{\rm i}q)$ is the matrix element of the electron–impurity interaction, where $s = -1$ for charged remote impurities, $d_{\rm i}$ is the spacing between the 2DES and the δ-doped layer, and $s = 0$ and $d_{\rm i} = 0$ for residual impurities immediately in the 2DES. The functions characterizing the overlap of the electron initial and final states are $Q_{N,N'}(\eta) = P_N^{(N'-N)}(\eta)\exp(-\eta/2)$, $P_N^{(N'-N)}(\eta) \propto L_N^{(N'-N)}(\eta)$, where $L_N^\Lambda(\eta)$ is the Laguerre polynomial. The LL form-factor is determined by the function $\delta(\varepsilon)$, which at a small broadening Γ can be assumed to be the Dirac delta function. The effect of microwave radiation is reduced to the inclusion of the energy of really absorbed or emitted M photons $M\hbar\Omega$ in the transition energy balance (in the argument of the function δ in equation (2)) and the appearance of the Bessel functions $J_M(\xi_\Omega(q_x, q_y))$, that reflects the contribution of virtually absorbed and emitted photons [8, 9]. Here

$$\xi_\Omega(q_x, q_y) = \frac{e\mathcal{E}}{m}\frac{|q_x e_x + q_y e_y - \mathrm{i}(\Omega_c/\Omega)(q_x e_y - q_y e_x)|}{|\Omega_c^2 - \Omega^2|}, \tag{3}$$

where m is the electron effective mass. Generally, as follows from equation (3), $\xi_\Omega(q_x, q_y)$ depends on the polarization properties of the incident radiation. It is convenient to present $\xi_\Omega(q_x, q_y)$ in the form

$$\xi_\Omega(q_x, q_y) = \xi_\Omega \Pi(q_x/q, q_y/q) Lq. \tag{4}$$

Here $\xi_\Omega = \sqrt{\langle|\xi_\Omega(q_x, q_y)|^2\rangle}/Lq$ corresponds to nonpolarized radiation, so that $\xi_\Omega = (e\mathcal{E}\sqrt{\Omega_c^2+\Omega^2}/\sqrt{2}mL\Omega|\Omega_c^2 - \Omega^2|)$, and $\Pi(q_x/q, q_y/q)$ describes an anisotropy associated with the specific of the radiation polarization. The properties of this function are determined by the general properties of the photoconductivity tensor [34].

When ξ_Ω becomes of the order of unity, the amplitude of the Larmor orbit centre oscillation in the microwave field is about L. Taking into account that for the transitions from the LLs with $N \gg 1$, which play the main role in a 2DES with a large filling factor, one can put $P_N^\Lambda(\eta) \simeq J_\Lambda(2\sqrt{N\eta})$, the inter-LL contribution to the photocurrent can be presented in the following form:

$$j_{\rm ph}^{(\rm inter)} \propto \mathcal{N}_{\rm i} L^2 \sum_{N,\Lambda,M\geqslant 0} f_N(1 - f_{N+\Lambda}) \int \mathrm{d}q_x\, \mathrm{d}q_y\, q_y q^{2s} \exp(-2d_{\rm i}q - L^2q^2/2)$$
$$\times J_\Lambda^2(\sqrt{2N}Lq) J_M^2(\xi_\Omega Lq))\delta(M\hbar\Omega - \Lambda\hbar\Omega_c + eEL^2q_y), \tag{5}$$

where f_N is the Fermi distribution function.

3. Ohmic regime

At $E < E_b$, assuming for definiteness that $\delta(\omega) = \gamma/\pi(\omega^2+\gamma^2)$, where $\gamma = \Gamma/\Omega_c$, expanding the right-hand side of equation (5) over eEL, taking into account that $J_\Lambda^2(\sqrt{2N}Lq) \simeq \cos^2[\sqrt{2N}Lq - (2\Lambda+1)\pi/4]/(\pi\sqrt{2N}Lq)$ for large N, and integrating, we arrive at

$$j_{\rm ph}^{(\rm inter)} \propto E\Gamma \sum_{\Lambda,M} \frac{\Theta_\Lambda \mathcal{R}_M(\xi_\Omega)(\Lambda\Omega_c - M\Omega)}{[(\Lambda\Omega_c - M\Omega)^2 + \Gamma^2]^2}. \tag{6}$$

Here

$$\Theta_\Lambda = \sum_N f_N(1 - f_{N+\Lambda})/\sqrt{N} \tag{7}$$

determines the temperature dependence (very weak) of the photocurrent associated with the scattering mechanism under consideration, and dependence

$$\mathcal{R}_M(z) = 2\int_0^\infty \mathrm{d}x\, x^{2(s+1)} \exp(-\beta x - x^2/2) \int_0^{2\pi} \mathrm{d}\varphi\, \sin^2\varphi J_M^2(z\, x\, \Pi(\varphi)) \tag{8}$$

accounts for the specific of the scattering matrix elements. Here $\beta = 2d_{\rm i}/L$ and $\Pi(\varphi) = \sqrt{2/[(\Omega_c/\Omega)^2+1]}|e_x\cos\varphi + e_y\sin\varphi - \mathrm{i}(\Omega_c/\Omega)(e_y\cos\varphi - e_x\sin\varphi)|$. One can see that near the cyclotron resonance, $\Omega \simeq \Omega_c$, $\Pi(\varphi) \simeq 1$ disregarding the radiation polarization. If the ac electric field is parallel to the dc electric field, i.e., $e_x = 1$ and $e_y = 0$, for an arbitrary ratio (Ω_c/Ω), one obtains $\Pi(\varphi) = \sqrt{2/[(\Omega_c/\Omega)^2+1]}\sqrt{1+[(\Omega_c/\Omega)^2-1]\sin\varphi}$. When the ac and dc electric fields are directed perpendicular to each other ($e_x = 0$ and $e_y = 1$), one obtains $\Pi(\varphi) = \sqrt{2/[1+(\Omega_c/\Omega)^2]}\sqrt{1+[(\Omega_c/\Omega)^2-1]\cos\varphi}$. In the case of circular polarization or when the microwave radiation is nonpolarized, we have $\Pi(\varphi) = 1$ at any ratio Ω_c/Ω. As can be inferred from the above formulae, in the case of a linearly (or elliptically) polarized radiation, the argument of the Bessel function in equation (8) depends on φ. Consequently, the dissipative microwave photoconductivity (as well as the Hall component) can exhibit some polarization selectivity [8, 9, 32]. As follows from the above examples, such an effect is not very significant. Leaving the effect of radiation polarization for consideration elsewhere, we shall disregard this effect in the following, setting $\Pi(\varphi) = 1$, so that

$$\mathcal{R}_M(z) = \int_0^\infty \mathrm{d}x\, x^{2(s+1)} J_M^2(xz) \exp(-\beta x - x^2/2). \tag{9}$$

At $\beta < 1$, from equation (9) one can come to the following approximation:

$$\mathcal{R}_M(z) \simeq \exp(-z^2) I_M(z^2), \tag{10}$$

where $I_M(\eta)$ is the modified Bessel function. In particular, at $z < 1$ and at $z \gg 1$, function $\mathcal{R}_M(z)$ can be approximated as

$$\mathcal{R}_M(z) \simeq \frac{\exp(-z^2) z^{2M}}{2^M M!}\left[1 + \frac{z^4}{4(M+1)}\right], \qquad \mathcal{R}_M(z) \simeq \frac{1}{\sqrt{2\pi z}}. \tag{11}$$

Considering that for the scattering mechanisms under consideration s varies from -1 to 0, we have put for simplicity $s = -1/2$.

Taking into account that $\mathcal{E}^2 \propto p_\Omega$, where p_Ω is the microwave power density, introducing the characteristic microwave power $\bar{p}_\Omega = m\Omega^3/2\pi\alpha \simeq 21.8 m\Omega^3 z_0^2$, where $\alpha = e^2/\hbar c \simeq 1/137$ and c is the speed of light, and using equations (6) and (9), for a 2DES with $\beta < 1$ we arrive at

$$j_{\mathrm{ph}}^{(\mathrm{inter})} \propto E\Gamma \exp\left[-Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right]\left\{I_1\left(Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right)\sum_\Lambda \frac{\Theta_\Lambda(\Lambda\Omega_{\mathrm{c}} - \Omega)}{[(\Lambda\Omega_{\mathrm{c}} - \Omega)^2 + \Gamma^2]^2}\right.$$
$$\left. + I_2\left(Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right)\sum_\Lambda \frac{\Theta_\Lambda(\Lambda\Omega_{\mathrm{c}} - 2\Omega)}{[(\Lambda\Omega_{\mathrm{c}} - 2\Omega)^2 + \Gamma^2]^2} + \cdots\right\}, \tag{12}$$

where $P = p_\Omega/\bar{p}_\Omega$ is the normalized microwave power and $f(\omega) = \omega(1+\omega^2)/(1-\omega^2)^2$. In particular, at $Pf(\Omega/\Omega_{\mathrm{c}}) < 1$, equation (12) can be presented as

$$j_{\mathrm{ph}}^{(\mathrm{inter})} \propto E\Gamma \exp[-Pf(\Omega/\Omega_{\mathrm{c}})]\left\{\frac{P}{2} f\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\sum_\Lambda \frac{\Theta_\Lambda(\Lambda\Omega_{\mathrm{c}} - \Omega)}{[(\Lambda\Omega_{\mathrm{c}} - \Omega)^2 + \Gamma^2]^2}\right.$$
$$\left. + \frac{P^2}{8} f^2\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\sum_\Lambda \frac{\Theta_\Lambda(\Lambda\Omega_{\mathrm{c}} - 2\Omega)}{[(\Lambda\Omega_{\mathrm{c}} - 2\Omega)^2 + \Gamma^2]^2} + \cdots\right\}. \tag{13}$$

One can see from the first term in the right-hand side of equations (12) and (13) that $j_{\mathrm{ph}}^{(\mathrm{inter})} = 0$ at $\Lambda\Omega_{\mathrm{c}} = \Omega$, whereas $j_{\mathrm{ph}}^{(\mathrm{inter})} > 0$ if $\Lambda\Omega_{\mathrm{c}}$ slightly exceeds Ω and $j_{\mathrm{ph}}^{(\mathrm{inter})} < 0$ (i.e., the dissipative microwave photocurrent is in opposition to the electric field) when $\Lambda\Omega_{\mathrm{c}}$ is somewhat smaller than Ω. As also follows from equations (12) and (13), $j_{\mathrm{ph}}^{(\mathrm{inter})}$ exhibits maxima and minima at $\Omega/\Omega_{\mathrm{c}} = \Lambda - \delta^{(+)}$ and $\Lambda + \delta^{(-)}$, respectively, where $0 < \delta^{(+)}, \delta^{(-)} < 1$. These maxima and minima correspond to the electron transitions with the absorption of one real photon. Apart from the single-photon maxima and minima, there are the maxima and minima associated with the two-photon (described by the second terms in the right-hand sides of equations (12) and (13)) and multiple-photon absorption processes.

Using equations (12) or (13), one can find that $\delta^{(\pm)} \simeq \sqrt{3}\Gamma/\hbar\Omega_{\mathrm{c}}$. At moderate microwave powers, estimating the LL broadening as (see, for example, [29, 35, 36]) $\Gamma = \sqrt{2\Omega_{\mathrm{c}} a/\pi\tau}$, where τ is the electron momentum relaxation time estimated from the electron mobility at $H = 0$ and $p_\Omega = 0$ and $a \geqslant 1$ is a semi-empirical broadening parameter which describes the difference between the scattering time τ and the net scattering time determined by all other

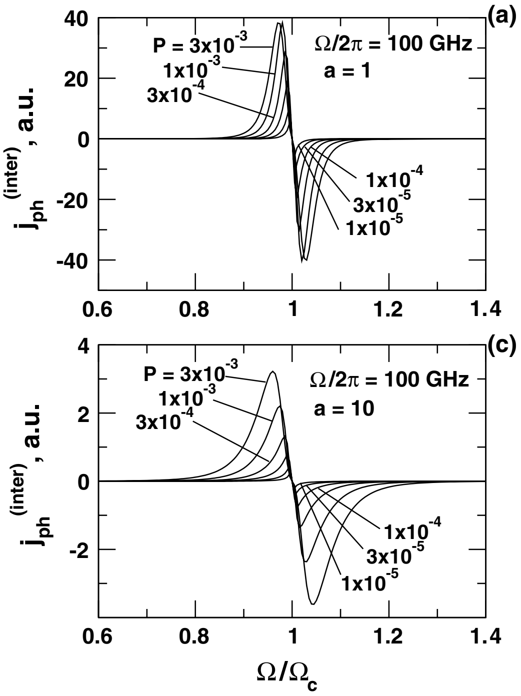

Figure 1. Dissipative photocurrent component $j_{\rm ph}^{\rm (inter)}$ versus inverse cyclotron frequency $\Omega/\Omega_{\rm c}$ in the vicinity of the cyclotron resonance at different normalized microwave powers P: (a) $\Omega/2\pi=$ 100 GHz, $a=1$, (b) $\Omega/2\pi=50$ GHz, $a=1$, (c) $\Omega/2\pi=100$ GHz, $a=10$, and (d) $\Omega/2\pi=50$ GHz, $a=10$.

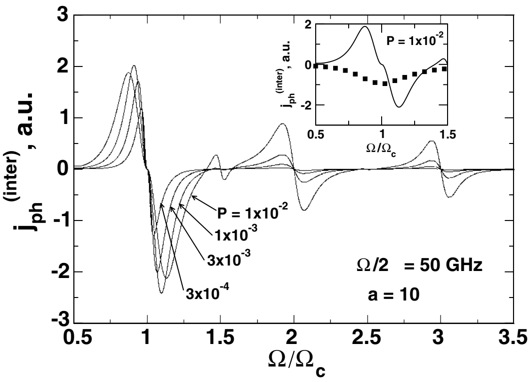

Figure 2. Oscillations of $j_{\rm ph}^{\rm (inter)}$ as a function of inverse cyclotron frequency $\Omega/\Omega_{\rm c}$ at different microwave powers ($\Omega/2\pi=50$ GHz, $a=10$). The inset shows $j_{\rm ph}^{\rm (inter)}$ (solid curve) and $j_{\rm ph}^{\rm (intra)}$ (squares) versus inverse cyclotron frequency.

mechanisms, one can find that $\delta^{(+)}\simeq\delta^{(-)}\simeq\sqrt{6a/\pi\Omega_{\rm c}\tau}$. Assuming $\tau=5.8\times10^{-10}$ s, $a=1-10$, and $\Omega/2\pi=50$ GHz (as in [23]) and using the above analytical estimate, we obtain $\delta^{(+)}\simeq\delta^{(-)}\simeq0.1-0.3$.

Figures 1 and 2 show the dependences of the inter-LL component of the microwave photocurrent $j_{\rm ph}^{\rm (inter)}$ on the inverse cyclotron frequency $\Omega/\Omega_{\rm c}$ for different broadening

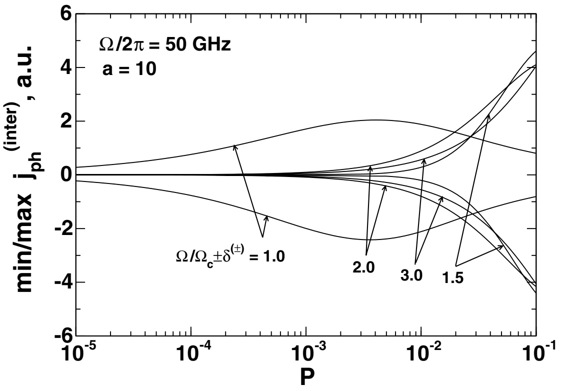

Figure 3. Maxima and minima of $j_{\rm ph}^{\rm (inter)}$ corresponding to different resonances as functions of normalized microwave power P ($\Omega/2\pi = 50$ GHz and $a = 10$).

parameters and for different microwave powers and frequencies calculated for a 2DES with $\beta < 1$ using equation (12). It is assumed that $\tau = 5.8 \times 10^{-10}$ s. The oscillatory $j_{\rm ph}^{\rm (inter)}$ versus $\Omega/\Omega_{\rm c}$ dependence at different P with the paired maxima and minima corresponding to the cyclotron resonance and its harmonics is shown in figure 2. One can see from figures 1 and 2 that the height of the maximum and the depth of the minimum near the cyclotron resonance (as well as near the cyclotron harmonics) increase with increasing microwave power. This increase is rather slow; the span of the photoconductivity as a function of the microwave power at low and moderate powers behaves similarly to a logarithmic function. However, at large powers, the height of the maximum (depth of the minimum) saturates and begins to fall (see figure 3). Figure 3 demonstrates the variations of the photoconductivity maxima and minima with increasing microwave power. As seen from figure 3, the height of the first one-photon maximum (depth of the relevant minimum), corresponding to $\Lambda = 1$ and $M = 1$, falls when the microwave power increases beyond some threshold value. In this range of microwave power, the maxima (minima) corresponding to higher resonances can be comparable with that near the cyclotron resonance. At high microwave powers, the two-photon resonant maxima (minima) can increase faster than those associated with the one-photon absorption processes (compare the curve for $\Omega/\Omega_{\rm c} \pm \delta^{(\pm)} = 1.5$ and the curves for $\Omega/\Omega_{\rm c} \pm \delta^{(\pm)} = 1$ and 2). In particular, when approximation (10) is valid, the ratio of the two-photon maxima $j_{\rm ph}^{(2,3)}$ ($M = 2$ and $\Lambda = 3$) to the single-photon maxima $j_{\rm ph}^{(1,2)}$ ($M = 1$ and $\Lambda = 2$) calculated using equation (12) can be estimated as $\max j_{\rm ph}^{(2,3)}/\max j_{\rm ph}^{(1,2)} \simeq 3.37 P e^{2P}$. As follows from the latter estimate, the ratio in question, being rather small at small P, markedly rises with increasing P. Figure 4 shows the variations of the position of the photoconductivity maxima near the cyclotron resonance $\delta^{(+)}$ and its width at half-maximum $\Delta^{(+)}$ (normalized by $\Omega_{\rm c}$) with increasing normalized microwave power P. One can also see from figures 1, 2, and 4 that the width of the resonant maximum and minimum increases with increasing microwave power. The broadening of the cyclotron absorption line at heightened microwave power associated with a similar mechanism was discussed recently [36]. Apart from this, at high microwave powers, the transition rate and, therefore, the LL broadening become larger. This can lead to an increase in the broadening parameter a and, hence, to an extra increase in the maximum and minimum width when the microwave power increases. At rather high powers, the two-photon resonances can be observable. Figure 2 exhibits a comparably weak maximum and minimum in the vicinity of $\Omega/\Omega_{\rm c} = 1.5$ corresponding to the two-photon transition ($M = 2$ and $\Lambda = 3$).

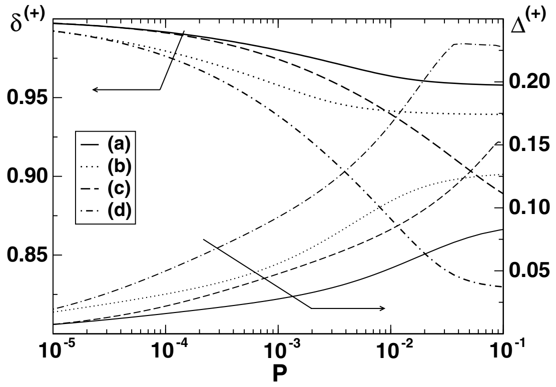

Figure 4. Position $\delta^{(+)}$ and width at half maximum $\Delta^{(+)}$ of near cyclotron maximum versus normalized microwave power P. Curves (a)–(d) correspond to photocurrent spectral dependences in figures 1(a)–(d).

These results are consistent with experimental data by Mani *et al* [20, 23] and Zudov *et al* [21] (see also [37]). Assuming, as in [21], that the power of the microwave source and the sample cross-section are 10–20 mW and 0.25 cm^{-2}, respectively, and using the above formula for the characteristic microwave power $\bar{p}_\Omega$, for $\Omega/2\pi = 50$ GHz one can find that the experimental conditions correspond to max $P \leqslant (1-2)\times 10^{-2}$. The later values are on the order of or larger than those used in figures 1 and 2.

As follows from equations (6) and (9), the height of the maxima and the depth of the minima determined by the function $\mathcal{R}_M(z)$ depend on the parameter β, i.e., on the thickness of the spacer separating the 2DES and the donor layer. This function is plotted in figure 5 for $M=1$ (one-photon absorption) and $M=2$ (two-photon absorption) and $z_0=1$. One can see that $\mathcal{R}_1(z)$ and $\mathcal{R}_2(z)$ pronouncedly decrease with increasing β. If $\beta \gg 1$, using equations (6) and (8), one can obtain instead of equation (12)

$$j_{\rm ph}^{\rm (inter)} \propto \frac{E\Gamma}{\beta}\left\{\mathcal{F}_1\left(\frac{\sqrt{P}}{\beta}\sqrt{f\left(\frac{\Omega}{\Omega_{\rm c}}\right)}\right)\sum_{\Lambda}\frac{\Theta_\Lambda(\Lambda\Omega_{\rm c}-\Omega)}{[(\Lambda\Omega_{\rm c}-\Omega)^2+\Gamma^2]^2}\right.$$
$$\left. + \mathcal{F}_2\left(\frac{\sqrt{P}}{\beta}\sqrt{f\left(\frac{\Omega}{\Omega_{\rm c}}\right)}\right)\sum_{\Lambda}\frac{\Theta_\Lambda(\Lambda\Omega_{\rm c}-2\Omega)}{[(\Lambda\Omega_{\rm c}-2\Omega)^2+\Gamma^2]^2}+\cdots\right\}. \tag{14}$$

Here for $s=-1$ and $\beta \gg 1$

$$\mathcal{F}_M(z) = \int_0^\infty {\rm d}x\, J_M^2(xz/z_0)\exp(-x). \tag{15}$$

At not too large z, setting $J_M(z) \simeq a_M \sin(\pi z/b_M)$, where a_M is the first maximum of the pertinent Bessel function and b_M corresponds to its first zero, one can obtain

$$\mathcal{F}_M(z) \simeq \left(\frac{\pi a_M}{2b_M}\right)^2 \frac{(z/z_0)^2}{[1+(\pi/b_M)^2(z/z_0)^2]}. \tag{16}$$

In particular, for $M=1$ equation (16) yields $F_1(z) \propto z^2/\beta(1+0.67z^2)$. At very large z, $F_M(z)$ becomes a decreasing function of z similar to that given by equation (11). Considering equation (16), near the first one-photon resonance ($\Lambda = 1$ and $M=1$) at $\beta \gg 1$, from equation (14) we obtain

$$j_{\rm ph}^{\rm (inter)} \propto \frac{E\Gamma P}{\beta^3} f\left(\frac{\Omega}{\Omega_{\rm c}}\right)\left[1+0.67\frac{P}{\beta^2}f\left(\frac{\Omega}{\Omega_{\rm c}}\right)\right]^{-1}\frac{(\Omega_{\rm c}-\Omega)}{[(\Omega_{\rm c}-\Omega)^2+\Gamma^2]^2}. \tag{17}$$

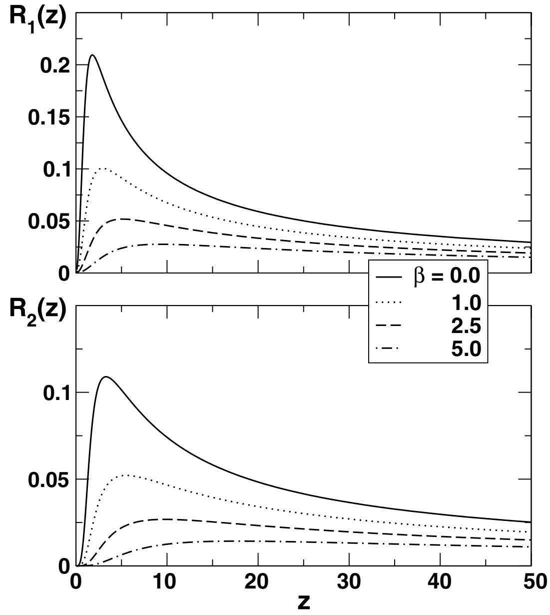

Figure 5. Functions $\mathcal{R}_1(z)$ and $\mathcal{R}_2(z)$ for different values of parameter β.

Equation (14) demonstrates a decrease in the microwave photoconductivity and the height of its maxima (depth of the minima) with increasing parameter β and slowing down with increasing power. The former is due to a decrease in the impurity scattering rate because of a smoothening of the fluctuating electric field created by charged impurities when the spacer becomes thicker (d_{i} becomes larger). One can see a similarity between the roles of the inverse parameter β^2 and the normalized microwave power P.

4. Photoconductivity in strong electric field

The average electric field $\bar{E}$ in the experimental situations [20–23] is rather moderate, so one can assume that $\bar{E} < E_{\mathrm{b}}$. However, under the condition of ANC, electric-field domain structures can be formed [25, 27, 33]. The value of the electric field E_0, at which the net dissipative conductivity changes its sign from negative to positive, essentially affects the magnitude of the electric field variations in the domain structures in question. Although it is unclear as yet what mechanism determines the threshold electric field E_0, one can assume that it can be much larger than E_{b}. In this case, the electric field in some regions may significantly exceed E_{b}. As shown by Shchamkhalova *et al* [38], the LL broadening $\hbar\Gamma$ and, therefore, the characteristic field E_{b} can substantially decrease in the electric field. Moreover, recent experiments [39] showed that strong long range ($\lambda \gg L$) fluctuations are present in 2DESs with high electron mobility. Due to these fluctuations, the local electric field can be of a rather large magnitude (about 30–150 kV cm^{-1}, as estimated by Kawano *et al* [18]), tangibly affecting the electron inter-LL transitions [1, 4, 13, 17]. Hence, the inequalities $E > E_{\mathrm{b}}$ or $E \gg E_{\mathrm{b}}$ can take place in the 2DES under consideration.

At relatively large net dc electric fields $E > E_{\mathrm{b}}$, $\delta(\omega)$ can be considered as the Dirac δ-function. Taking into account that the transitions between high LLs provide the main contribution to the mechanism of microwave photoconductivity under consideration, and integrating over q_y in equation (5), we obtain

$$j_{\rm ph}^{\rm (inter)} \propto \sum_{\Lambda,M} \Theta_\Lambda \mathcal{R}_M\left(Pf\left(\frac{\Omega}{\Omega_{\rm c}}\right), \frac{\hbar|\Lambda\Omega_{\rm c} - M\Omega|}{eEL}\right)\left[\frac{\hbar(\Lambda\Omega_{\rm c} - M\Omega)}{eE|E|L}\right], \quad (18)$$

where

$$\mathcal{R}_M(z, y) = \int_0^\infty {\rm d}x\, J_M^2(z\sqrt{x^2+y^2})\frac{\exp[-\beta\sqrt{x^2+y^2} - (x^2+y^2)/2]}{(x^2+y^2)^{1/2-s}}. \quad (19)$$

If $\beta < 1$, the y-dependence of $\mathcal{R}_M(z, y)$ is mainly given by the factor $\exp(-y^2/2)$. Considering this, from equation (19) we obtain

$$j_{\rm ph}^{\rm (inter)} \propto \sum_{\Lambda,M} \Theta_\Lambda \mathcal{R}_M^*\left(Pf\left(\frac{\Omega}{\Omega_{\rm c}}\right), \frac{\hbar|\Lambda\Omega_{\rm c} - M\Omega|}{eEL}\right) \times \left[\frac{\hbar(\Lambda\Omega_{\rm c} - M\Omega)}{eE|E|L}\right]\exp\left[-\frac{\hbar^2(\Lambda\Omega_{\rm c} - M\Omega)^2}{2(eEL)^2}\right]. \quad (20)$$

Here

$$\mathcal{R}_M^*(z, y) = \int_0^\infty {\rm d}x\, J_M^2\big(z\sqrt{x^2+y^2}\big)\frac{\exp(-x^2/2)}{(x^2+y^2)^{1/2-s}} \quad (21)$$

is a rather smooth function of y. At small microwave powers, equations (18) and (20) coincide with that obtained a long time ago [4]. As follows from equation (18), the microwave conductivity in a strong electric field also exhibits an oscillatory behaviour (associated primarily with the two last factors in the right-hand side of equation (20)) with maxima and minima at $\Omega/\Omega_{\rm c} = \Lambda - \delta^{(+)}$ and $\Omega/\Omega_{\rm c} = \Lambda + \delta^{(-)}$, respectively, for the one-photon transitions, and $\Omega/\Omega_{\rm c} = [\Lambda - \delta^{(+)}]/M$ and at $\Omega/\Omega_{\rm c} = [\Lambda + \delta^{(-)}]/M$ for the transitions with absorption of M photons. Here $\delta^{(+)} \simeq \delta^{(-)} \simeq eEL/\hbar\Omega_{\rm c} = E/E_{\rm c}$. The width of the maxima and minima in question linearly increases with E as $\Delta^{(+)} \simeq \Delta^{(-)} \simeq 2.24E/E_{\rm c}$, while their height is proportional to E^{-1}. Indeed, equation (20) yields the following dependence of the $\min/\max j_{\rm ph}^{\rm (inter)}$ for the one-photon absorption near the cyclotron resonance on the microwave power P:

$$\min/\max j_{\rm ph}^{\rm (inter)} \propto \frac{\exp(-Pf^{(\mp)})}{E} I_1(Pf^{(\mp)}), \quad (22)$$

where $f^{(\pm)} = f(1 \pm E/E_{\rm c})$. Here, assuming $s = -1$ and $\beta < 1$, we have used the following estimate: $\mathcal{R}_1^*(z, 1) \simeq 0.4\exp(-z^2)I_1(z^2)$. Figure 6 shows the dependences of $\min/\max j_{\rm ph}^{\rm (inter)}$ on the electric field at different microwave powers calculated using an interpolation formula leading to $\min/\max j_{\rm ph}^{\rm (inter)}$ following from equation (12) in the low-field region (for $\Omega/2\pi = 100$ GHz and $a = 1$ and 10) and to equation (22) in the high-field region.

At $\beta \gg 1$ and $s = -1$,

$$\mathcal{R}_1(z, y) \simeq \frac{z^2}{4}\int_0^\infty {\rm d}x\, \frac{\exp(-\beta\sqrt{x^2+y^2})}{(x^2+y^2)^{1/2}} = \frac{z^2}{4}K_0(\beta y), \quad (23)$$

where $K_0(\beta y)$ is the McDonald function. Taking this into account, at low microwave powers we arrive at

$$j_{\rm ph}^{\rm (inter)} \propto Pf\left(\frac{\Omega}{\Omega_{\rm c}}\right)\sum_\Lambda \Theta_\Lambda\left[\frac{\hbar(\Lambda\Omega_{\rm c} - \Omega)}{eE|E|L}\right]K_0\left(\frac{H|\Lambda\Omega_{\rm c} - \Omega|d_{\rm i}}{cE}\right). \quad (24)$$

It is instructive that in this particular case ($\beta \gg 1$, i.e., $d_{\rm i} \gg L$), the argument of the McDonald function in equation (24), which can be presented as $|\Lambda\Omega_{\rm c} - \Omega|d_{\rm i}/v_{\rm H}$, where $v_{\rm H} = cE/H$ is the Hall drift velocity, is determined by classical values (it does not include the Planck constant).

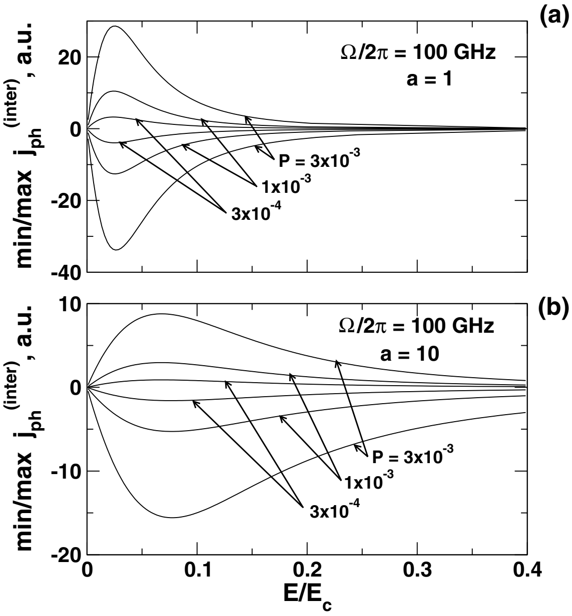

Figure 6. Near cyclotron maxima and minima of $j_{\mathrm{ph}}^{(\mathrm{inter})}$ versus electric field at different microwave powers for $\Omega/2\pi = 100$ GHz and different broadening parameters a: (a) $a = 1$ and (b) $a = 10$.

A marked sensitivity of the resonant maxima and minima to the electric field can be crucial (together with the LL-broadening associated with the interactions other than the impurity scattering) for the explanation of their fairly wide width observed experimentally, particularly, considering strong electric-field fluctuations [39].

5. Suppression of intra-LL photoconductivity

Taking into account that, as follows from equation (2), the probability of the elastic impurity scattering in the presence of microwave radiation differs from that in its absence by the factor $J_0^2(\xi_\Omega(q_x, q_y))$, and generalizing the results of [2] (see also [36]), the dissipative current associated with the intra-LL impurity scattering can given by

$$j^{(\mathrm{intra})} \propto E\mathcal{N}_{\mathrm{i}}L^2 \sum_N f_N(1 - f_N) \times \int \mathrm{d}q_x\,\mathrm{d}q_y\,q_y^2 q^{2s} \exp(-2d_{\mathrm{i}}q - L^2q^2/2) J_0^2\big(\sqrt{2N}Lq\big) J_0^2(\xi_\Omega Lq)) \qquad (25)$$

at $E < E_{\mathrm{b}}$, and

$$j^{(\mathrm{intra})} \propto \frac{\mathcal{N}_{\mathrm{i}}}{E} \sum_N f_N(1 - f_N) \times \int \mathrm{d}q_x\,\mathrm{d}q_y\,q^{2s} \exp(-2d_{\mathrm{i}}q - L^2q^22) J_0^2\big(\sqrt{2N}Lq\big) J_0^2(\xi_\Omega Lq)) \qquad (26)$$

at $E \gg E_{\mathrm{b}}$. Using the same procedure as in section 3, equation (25) for the case $\beta < 1$ and $E < E_{\mathrm{b}}$ can be reduced to

$$j^{(\mathrm{intra})} \simeq j_{\mathrm{dark}}^{(\mathrm{intra})}\mathcal{R}_0(\xi_\Omega) \simeq j_{\mathrm{dark}}^{(\mathrm{intra})} \exp\left[-Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right] I_0\left(Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right), \qquad (27)$$

where $j_{\mathrm{dark}}^{(\mathrm{intra})} = j^{(\mathrm{intra})}|_{P=0}$ is the dissipative current associated with the intra-LL impurity scattering without irradiation (dark current). Using equation (23), we arrive at the following expression for the photoconductivity associated with the microwave-stimulated variation of the dissipative intra-LL conductivity:

$$j_{\mathrm{ph}}^{(\mathrm{intra})} \simeq j_{\mathrm{dark}}^{(\mathrm{intra})}\left\{\exp\left[-Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right]I_0\left(Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right)\right)-1\right\} \simeq -\frac{3}{4}j_{\mathrm{dark}}^{(\mathrm{intra})}Pf\left(\frac{\Omega}{\Omega_{\mathrm{c}}}\right). \qquad (28)$$

The last term in the right-hand side of equation (24) is valid when $Pf(\Omega/\Omega_{\mathrm{c}}) < 1$. As seen from equation (24), the contribution of the effect of microwave radiation on the elastic intra-LL impurity scattering to the photoconductivity is negative. This contribution as a function of $\Omega/\Omega_{\mathrm{c}}$ exhibits a smeared minimum at the cyclotron resonance. This minimum is attributed to the suppression of the impurity scattering by microwave radiation. Such a suppression (associated with the absorption and emission of virtual phonons) is most effective when the amplitude of the Larmor orbit centre oscillation is maximum, i.e., at the cyclotron resonance. The $j_{\mathrm{ph}}^{(\mathrm{intra})}$ versus $\Omega/\Omega_{\mathrm{c}}$ dependence calculated using equation (24) is shown by squares in the inset in figure 2. Since this dependence does not comprise any resonant factor, it cannot affect significantly the oscillatory dependences associated with the photon-assisted inter-LL transitions, although it leads to some deepening of the photoconductivity minimum near the cyclotron resonance. Formulae (23) and (24) correspond to the total suppression of the dissipative intra-LL conductivity, i.e. to $j^{(\mathrm{intra})} = 0$ and, therefore, $j_{\mathrm{ph}}^{(\mathrm{intra})} = -j_{\mathrm{dark}}^{(\mathrm{intra})}$, at the cyclotron resonance. However, the dephasing of the electron oscillatory movement in the microwave field due to different scattering events can limit the suppression of the dissipative intra-LL conductivity. This effect can be included by the substitution of the function $f^*(\omega) = f(\omega)(1-\omega^2)/[(1-\omega)^2+\gamma^2]$ for the function $f(\omega)$ introduced in section 3. Taking into account that $f^*(1) = 1/2\gamma^2$, at $P < 2(\Gamma/\Omega_{\mathrm{c}})^2$ we obtain $\min j^{(\mathrm{intra})} \simeq j_{\mathrm{dark}}^{(\mathrm{intra})}[1 - P(\Omega_{\mathrm{c}}/\sqrt{2}\Gamma)^2]$ and, consequently, $\min j_{\mathrm{ph}}^{(\mathrm{intra})} \simeq -j_{\mathrm{dark}}^{(\mathrm{intra})}P(\Omega_{\mathrm{c}}/\sqrt{2}\Gamma)^2$. At $P \gg 2(\Gamma/\Omega_{\mathrm{c}})^2$, one obtains $\min j^{(\mathrm{intra})} \simeq j_{\mathrm{dark}}^{(\mathrm{intra})}(\Gamma/\sqrt{\pi P}\Omega_{\mathrm{c}})$ and $\min j_{\mathrm{ph}}^{(\mathrm{intra})} \simeq -j_{\mathrm{dark}}^{(\mathrm{intra})}[1 - (\Gamma/\sqrt{\pi P}\Omega_{\mathrm{c}})] \simeq -j_{\mathrm{dark}}^{(\mathrm{intra})}$.

6. Comments

The proposed model describes the following features of the microwave conductivity in a 2DES subjected to a magnetic field:

(i) The zeroth microwave conductivity at the cyclotron resonance and its harmonics, as well as the location of the photoconductivity maxima and minima in the vicinity of the resonances with ANC in the minima.
(ii) Nonlinear dependences of the photoconductivity on the microwave power characterized by slowing down, saturation, and even decrease in the minima/maxima magnitude with increasing microwave power.
(iii) Shift of the photoconductivity maxima and minima and their broadening with increasing microwave power and electric field.
(iv) Possibility of the suppression of the dissipative conductivity associated with the intra-LL transitions by the microwave radiation.

At sufficiently strong irradiation the negativity of the dissipative photoconductivity in its minima can lead to the zeroth or negative net dissipative conductivity. Since the sample resistance in the Hall bar geometry and the conductance in the Corbino geometry are virtually proportional to the net dissipative conductivity, the zeroth of the latter corresponds to the zeroth resistance or the zeroth conductance (depending on the configuration), and the oscillations of the dissipative photoconductivity give rise to oscillations of the characteristics of the sample as a whole. The photon-assisted scattering processes also result in some variation of the Hall component of the conductivity tensor (as considered in the case of 3DESs [9]). However, due to a large value of the classical Hall conductivity, the variations of the Hall conductivity induced by microwaves yield a relatively small contribution to the sample resistance.

The positions of the photoconductivity zeros, maxima, and minima are determined by the specific features of the photon-assisted impurity scattering of electrons in the magnetic field. Some of these features of the microwave photoconductivity, predicted theoretically [4, 7] and observed experimentally [21, 23] (see also [21, 22, 37]), have been discussed in the framework of different theoretical models [24, 26]. It is instructive that the effect of suppression of the impurity intra-LL scattering by microwave radiation can result in a shift of the microwave photoconductivity zero to Ω/Ω_c slightly smaller than unity. Thus, according to the above results, the phase shift of the photoconductivity maxima and minima is determined by different factors and it is not necessarily equal to $\pm\pi/4$ as stated in [20] (see also the discussion in [37]).

A similar mechanism of ANC stimulated by radiation (including the suppression of the transitions without absorption and emission of real photons) is also associated with the electron transitions between the spatially separated states observed experimentally in multiple-quantum-well structures with sequential resonant tunnelling some time ago [40].

The photon-assisted scattering on acoustic phonons can also lead to the oscillatory dependence of the dissipative conductivity with ANC at Ω/Ω_c between the cyclotron resonances and its harmonics [30, 31]. It is remarkable that this mechanism provides the photoconductivity minima and maxima at approximately the same values of Ω/Ω_c, where the photon-assisted impurity scattering yields, in contrast, maxima and minima. Thus, the photon-assisted acoustic scattering can to some extent suppress the oscillations associated with the photon-assisted impurity scattering. Such a competition of the mechanisms in question can be essential because the piezoelectric acoustic scattering is one of the main scattering mechanisms limiting the electron mobility in perfect 2DESs at low temperatures [41]. The contribution of the photon-assisted acoustic scattering mechanism to the microwave conductivity markedly increases with the temperature, particularly in the range where T is comparable with the characteristic energy of acoustic phonons $\hbar s/L$ (s is the speed of sound). Assuming $s = 3\times 10^5$ and $H = 1-2$ kG, one can obtain $T_{ac} \simeq 0.4$ K. The latter value is only slightly smaller than T in the experiments. Hence, taking into account that the photocurrent associated with the photon-assisted impurity scattering exhibits a very weak temperature dependence (given by the factor Θ_Λ), the suppression of the microwave photoconductivity oscillations and the effect of ANC (a pronounced decrease in the minima/maxima magnitude) with increasing temperature observed experimentally [20, 21] can possibly be attributed to the inclusion of the above mentioned photon-assisted acoustic scattering mechanism. This mechanism, however, provides a fairly slow decrease in the magnitudes of photoconductivity maxima and minima with increasing temperature. One can set up the hypothesis that relatively small temperature variations of the photoconductivity lead to a significant change in the electric-field distributions whose shape determines the values of the sample resistance and conductance. Indeed, the interference of the photon-assisted impurity and phonon mechanisms results in the transformation of the net current–voltage characteristics. Even small temperature variations of the net conductivity (near its zero) can lead to a transition from a uniform electric field distribution to a domain structure with different sample resistance (conductance) due to an instability [11]. Apart from this, temperature variations can result in a significant change in the value of the dc electric field E_0 at which the net dissipative current turns zero and, hence, strongly affect the domain shape and the observable characteristics [25, 27], in particular, the residual resistance [42, 43].

The photoconductivity considered above (and in [4, 7, 26, 28, 29, 32]) is associated with a direct effect of the microwave ac field on the impurity scattering of electrons. However, the photon-assisted impurity scattering is accompanied by the absorption of photons and, hence, some heating of the 2DES. Such a heating can lead to a variation of the dissipative conductivity associated with the scattering on impurities and acoustic phonons. The dissipative current in strong magnetic fields associated with the intra-LL impurity scattering is sensitive to the temperature (mainly via the factor Θ_0). This effect in part gives rise to the smearing of the Shubnikov–de Haas oscillation with increasing temperature. However, the temperature dependence of Θ_0 in the range where the oscillations of microwave photoconductivity under consideration were observed experimentally is rather weak. The electron heating caused by the photon-assisted impurity scattering processes is compensated by the relaxation processes with the electron transitions from higher to lower LLs with the emission of acoustic phonons. As shown recently [44], these relaxation processes contribute to the dissipative conductivity (their contribution is negative), complicating the spectral dependence of the microwave photoconductivity.

The nontrivial dependences of the maxima/minima span on the microwave power are consistent with those observed experimentally [20, 23] and are attributed to the multi-photon effects including the absorption and emission of real and virtual phonons (see [8–10, 29, 35]). A relatively large width of the microwave photoconductivity maxima and minima is often identified with an extra broadening of LLs (in comparison with the LL broadening corresponding to the electron collision time determining the electron mobility) associated with more complex interaction discussed previously [24, 26, 29]. As pointed out above, this effect can be attributed to the nonlinear effects of the ac microwave field and strong electric-field dependences of the local photoconductivity characteristics combined with strong long range fluctuations of the electric field. In the presence of strong long range ($\lambda \gg L$) electric-field fluctuations the macroscopic properties of the 2DES, in particular the domain structures with the scale exceeding λ, can be determined by the components of the dissipative photocurrent averaged over these fluctuations $\overline{j_{\rm ph}(E)E_x/E}$ and $\overline{j_{\rm ph}(E)E_y/E}$. Due to a strong electric-field dependence of $j_{\rm ph}$ (see equation (18)), the dependences of the averaged quantities on the components of the averaged electric field $\overline{E_x} \propto V$ and $\overline{E_y} \propto V_{\rm H}$ (where V and $V_{\rm H}$ are the potential drop along the current and the Hall current, respectively) as well as their spectral dependences can be significantly different from those of the local dissipative current.

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